UNIT-4

Knuckle Joint:

A Knuckle Joint is used to Joint two rods which are acted upon by tensile load.

-> It may be readily disconnected for adjustment or repair.

-> Steel or wrought iron is used for making the joint.

Applications:

-> Used in the links of cycle chains to connect different links

-> Tie rod joint for roof truss.

-> Tension link in bridge structure and lever.

-> Pump rod joint

-> Valve rod joint with eccentric rod.

Main parts of Knuckle Joint:

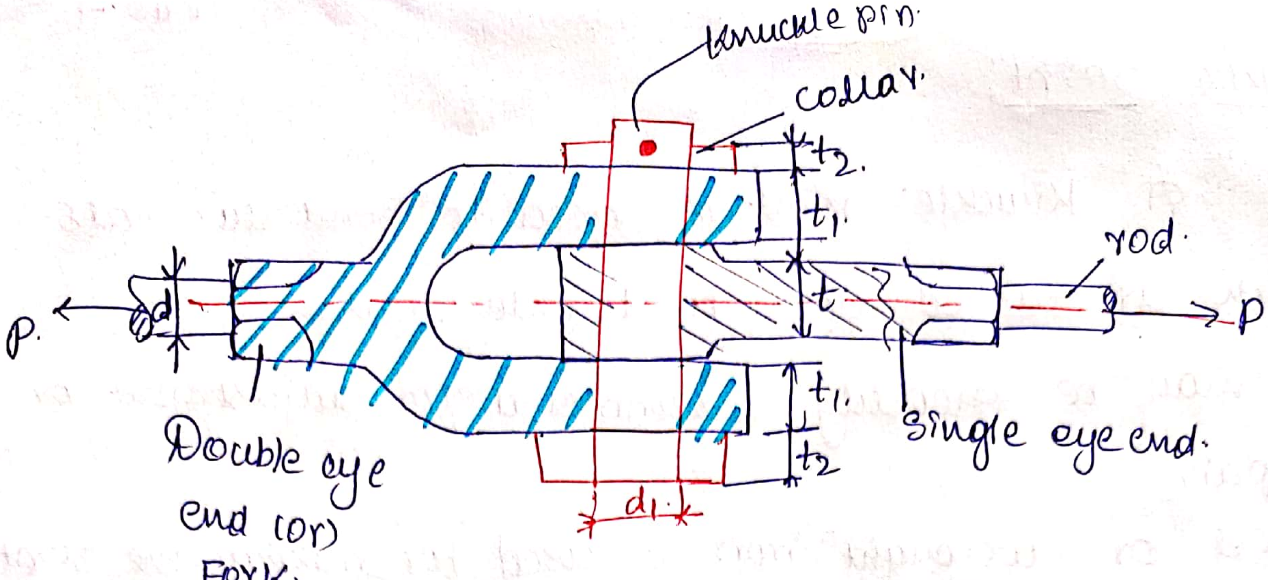
-> Single eye rod.

-> Fork (or) Double eye rod.

-> Knuckle pin.

-> Knuckle pin collar

-> Split pin.



P = Tensile load acting on the rod.

d = dia. of the rod.

d_1 = Dia. of pin.

d_2 = outer dia. of eye.

t = thickness of single eye.

t_1 = thickness of fork.

σ_t, τ & σ_c = permissible tensile, shear & crushing stresses.

Dimensions of various parts of the knuckle Joint from the nomenclature.

If d = dia. of the rod,

$\boxed{d_1 = d.}$ d_2 = outer dia of eye.

$\boxed{d_2 = 2d.}$

Dia. of knuckle pinhead & collar

$\boxed{d_3 = 1.5d.}$

thickness of single eye (or) rod end.

$\boxed{t = 1.25d.}$

∴ Thickness of fork, $\boxed{t_1 = 0.75d}$

" " Pin head, $\boxed{t_2 = 0.5d.}$

⇒ Design procedure of knuckle joint: (from the failures)

Step ①: Design of rod.

Step ②: Design of single eye-end.

(i) under tension

(ii) under ~~f~~ shear failure

(iii) " crushing

Step ③: Design of fork.

(i) failure by tension

(ii) " under shear

(iii) " " crushing.

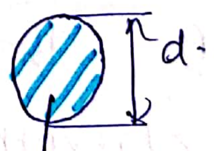
Step ④: Design of knuckle pin.

(i) failure by shear

(ii) " " Bending.

Step ①: Design of rod:

$A = \frac{\pi}{4} d^2$



Resisting Area.

$A = \frac{\pi}{4} d^2$

Considering failure of rod under Tension.

∴ Tensile strength of rod:

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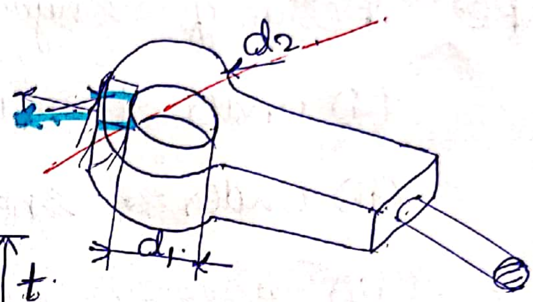
$$P = \sigma_t \times A$$

$$\boxed{P = \frac{\pi}{4} d^2 \times \sigma_t}$$

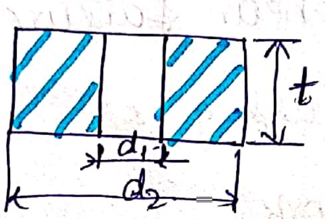
from this 'd' can be calculated.

Step ②: Design of single eye end.

(i) Considering Tensile failure:



Resisting Area,



$(A) = (d_2 - d_1)t$

Tensile strength of single eye end:

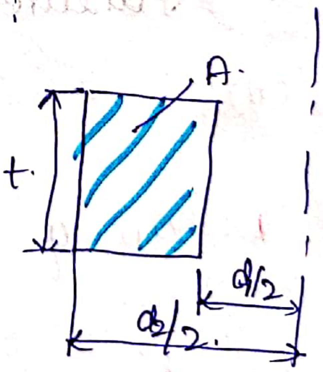
$$P = \sigma_t \times A$$

$$\boxed{P = (d_2 - d_1)t \times A}$$

from this d_2 (or) t we can calculate.

(ii) Considering shearing failure:

Resisting Area, (Double shear)



$$(A) = \frac{d_2 - d_1}{2} \times 2 \times t$$

$$A = (d_2 - d_1) \times t$$

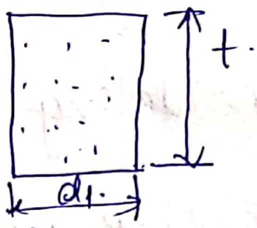
Shear strength of the single eye end.

$$P = A \times \tau$$

$$\boxed{P = (d_2 - d_1)t \times \tau}$$ → used for checking

(ii) considering crushing failure:

Resisting Area $(A) = d_1 \times t$

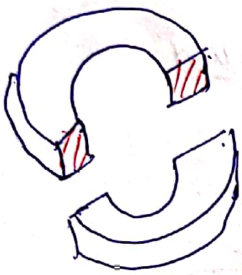


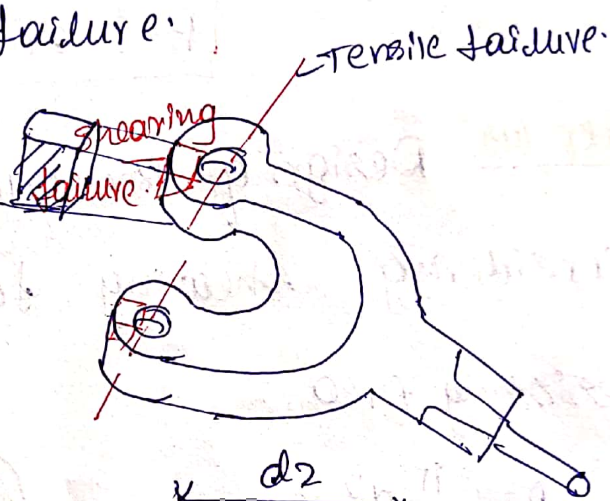
Crushing strength, $P = A \times \sigma_c$.

$$\boxed{P = d_1 \times t \times \sigma_c}$$ → σ_c is checked.

Step ③: Design of fork (or) Double end end.

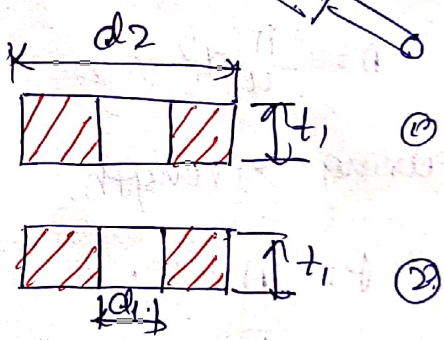
(i) considering tensile failure:





Resisting Area.

$(A) = (d_2 - d_1) \times t_1 \times 2$.



Tensile strength of fork.

$P = A \times \sigma_t$.

$$\boxed{P = (d_2 - d_1) \times t_1 \times 2 \times \sigma_t}$$

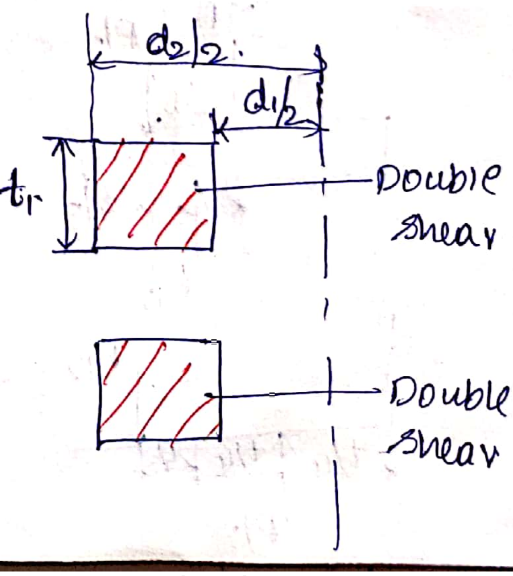
σ_t can be checked.

(ii) considering shear failure:

Resisting Area,

$A = \frac{d_2 - d_1}{\cancel{2}} \times t_1 \times \cancel{2} \times 2$

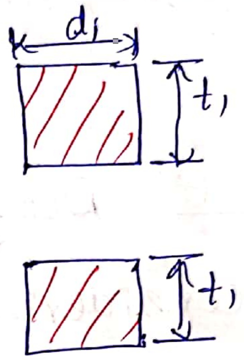
$A = d_2 - d_1 \times 2t_1$.



Shearing strength, $P = A \times \tau$.

$$\boxed{P = (d_2 - d_1) \times 2t_1 \times \tau}$$

(iii) considering crushing failure:



Resisting Area $A = d_1 \times t_1 \times 2$.

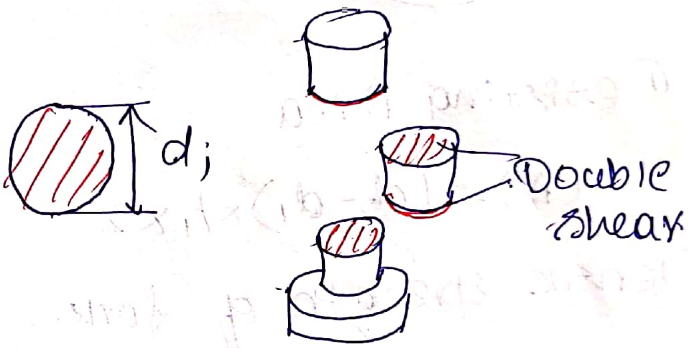
Crushing strength, $P = A \times \sigma_c$.

$$\boxed{P = d_1 \times 2t_1 \times \sigma_c}$$

Step (4): Design of knuckle pin. $\rightarrow \sigma_c$ checked.

(i) considering shearing failure.

Resisting Area,



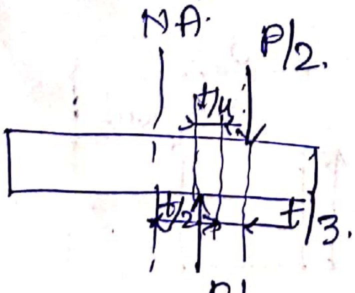
$$A = \frac{\pi}{4} d_1^2 \times 2.$$

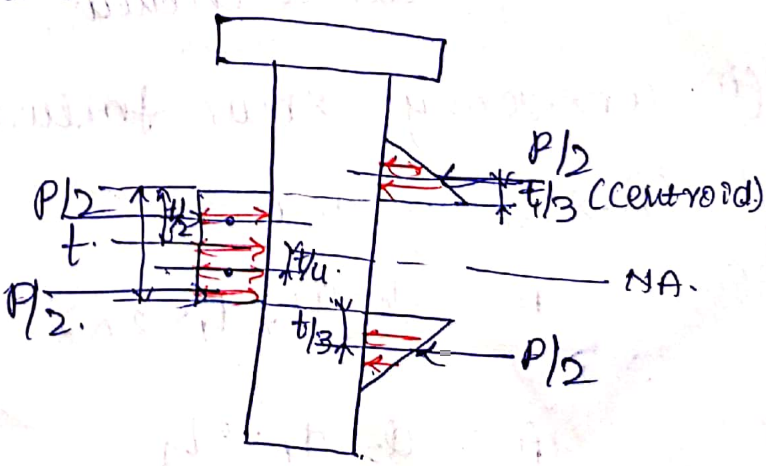
Shearing strength,

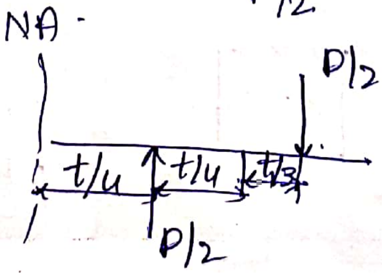
$$P = A \times \tau.$$

$$\boxed{P = \frac{\pi}{4} d_1^2 \times 2 \times \tau}$$ $\rightarrow d_1$ is calculated.

(ii) considering Bending failure.







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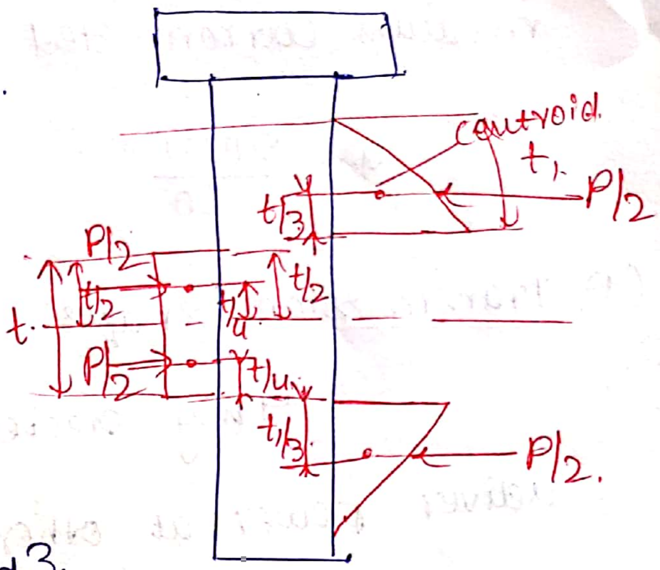
Taking moments about NA,

$$M = \frac{P}{2}\left(\frac{t_1}{3}+\frac{t}{2}\right) - \frac{P}{2}\times\frac{t}{4}$$

$$= \frac{P}{2}\left(\frac{t_1}{3}+\frac{t}{2}-\frac{t}{4}\right)$$

$$= \frac{P}{2}\left(\frac{t_1}{3}+\frac{2t-t}{4}\right)$$

$$M = \frac{P}{2}\left(\frac{t_1}{3}+\frac{t}{4}\right)$$



$$\sigma_b = \frac{M}{Z}, \quad Z = \frac{\pi}{32} d_1^3$$

$$\frac{t/2}{2} \Rightarrow \frac{t}{2}\times\frac{1}{2} = \frac{t}{4}$$

$$\therefore \sigma_b = \frac{P/2\left(\frac{t_1}{3}+\frac{t}{4}\right)}{\frac{\pi}{32}(d_1^3)}$$

→ Design of shafts:

Shafts:

Shaft is a rotating m/c element which either receives power, transmits power or both.

→ Shafts are of circular c/s.

→ Shafts may be solid or hollow depending upon the application.

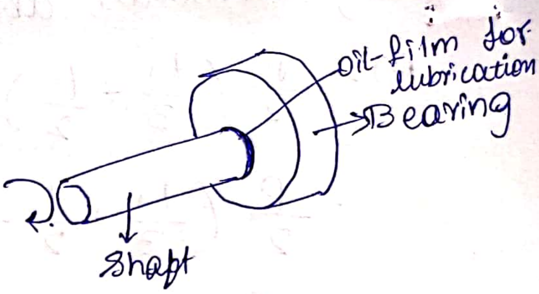
→ Shafts may be of two types

(i) Transmission shafts

(ii) m/c shafts

→ material is mostly steel (or) medium carbon steel.

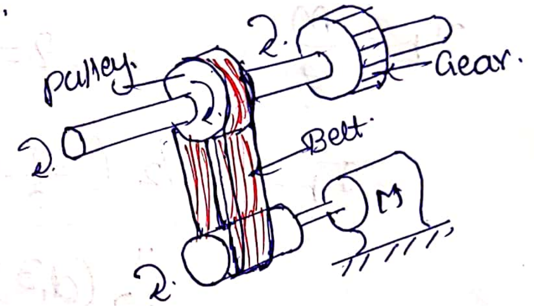
$$P = \frac{2\pi NT}{60}$$



(i) Transmission shafts:

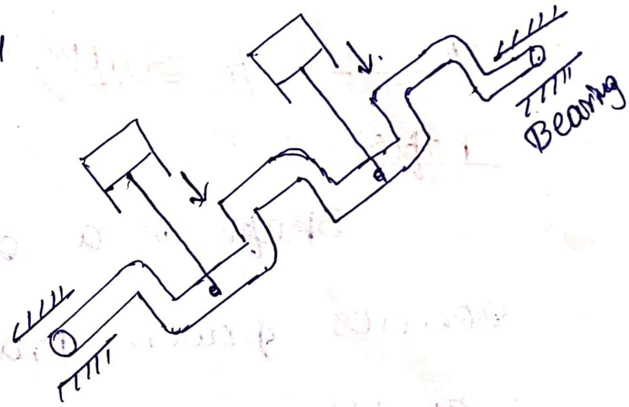
They recieve power from one end & deliver power at other end.

→ Transmission elements may pulleys, gears etc..



(ii) Machine - shafts:

They are an integral part of a m/c or also we can say that they are the main member in a m/c.



ex: Crank-shaft in I.C. Engines.

⟹ Design of shaft:

→ Design of shaft is to calculate the correct size of the shaft. i.e. shaft dia (d)

→ This is determined by two criterias

① Strength criteria → Based on permissible stresses

② Regidity criteria → Based on angular deformation
↓
Torsional.

→ Design of shafts Based on strength Basis:

↳ Based on permissible stresses.

→ We know that shaft will be subjected to axial loading, Bending loading and twisting moment also.

→ Axial loading → Axial stress $= \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{4P}{\pi d^2}$

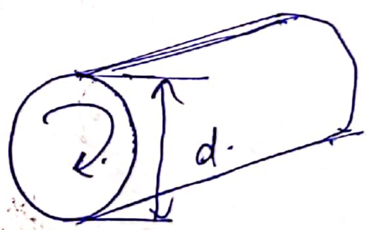
→ Bending moment → Bending stress, $\sigma_b = \frac{M}{Z}$

→ Twisting " ⇒ Shear stress, $\tau = \frac{Tr}{J}$

→ Combination.

→ Design of the shaft on the basis of Twisting moment only:

Torsion equation,



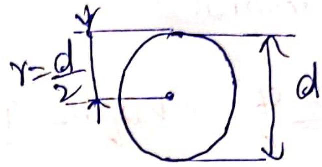
$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

T = Torque (or) twisting moment (or) Torsion (N-m)

T = Tangential force × radius of shaft.

J = Polar moment of inertia (mm^4)

for solid shaft,



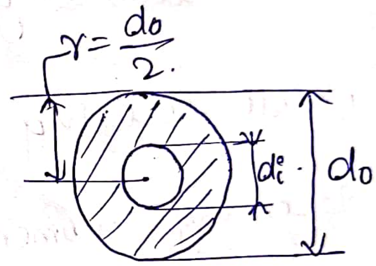
$$J = I_{xx} + I_{yy}$$

$$= \frac{\pi}{64} d^4 + \frac{\pi}{64} d^4$$

$$= \cancel{2} \times \frac{\pi}{\cancel{64}_{32}} d^4$$

$$= \frac{\pi}{32} d^4.$$

for hollow shaft!



$$J = \frac{\pi}{64}(d_o^4 - d_i^4) + \frac{\pi}{64}(d_o^4 - d_i^4)$$

$$= \cancel{2} \times \frac{\pi}{\cancel{64}_{32}}(d_o^4 - d_i^4)$$

$$= \frac{\pi}{32}(d_o^4 - d_i^4)$$

τ = Torsional shear stress N/mm^2.

r = radius of the shaft (dis from N.A to outermost layer)

$r = \frac{d}{2}$ (for solid shaft)

$r = \frac{d_o}{2}$ (for hollow shaft)

G = Modulus of rigidity (MPa) = $G \Rightarrow \frac{\text{shear stress}}{\text{shear strain}}$.

θ = Angle of twist (radian)

l = length of the shaft (mm)

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i) for solid shaft:

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = \frac{\tau \cdot J}{r}$$

$$T = \frac{\tau}{\cancel{d/2}} \times \frac{\pi}{\cancel{32}_{16}} d^{\cancel{4}\,3}$$

$$T = \frac{\pi}{16} d^3 \tau$$

$$16T = \pi \cdot \tau \cdot d^3$$

$$\boxed{T = \frac{\pi}{16} \cdot d^3 \, \tau}$$

from this eqn. we can find the dia (d)

(ii) for hollow shaft:

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = \frac{\tau \cdot J}{r}$$

$$T = \frac{\tau}{\frac{d_o}{2}} \cdot \frac{\pi}{32} (d_o^4 - d_i^4)$$

$$= \frac{\cancel{2}\tau}{d_o} \frac{\pi}{\cancel{32}_{16}} (d_o^4 \; d_i^4)$$

$$T = \frac{\pi}{16} \tau \cdot \left[\frac{d_o^4 - d_i^4}{d_o} \right].$$

$$= \frac{\pi}{16} \cdot \tau \cdot \frac{d_o^3}{d_o^3} \left(\frac{d_o^4 - d_i^4}{d_o} \right)$$

$$T = \frac{\pi}{16} \tau \cdot d_o^3 \left(\frac{d_o^4 - d_i^4}{d_o^4} \right)$$

$$= \frac{\pi}{16} \tau \cdot d_o^3 \left(\cancel{\frac{d_o^4}{d_o^4}} - \frac{d_i^4}{d_o^4} \right)$$

$$= \frac{\pi}{16} \cdot \tau \cdot d_o^3 \left(1 - \left(\frac{d_i}{d_o} \right)^4 \right)$$

$$\boxed{T = \frac{\pi}{16} \tau \cdot d_o^3 [1 - (k)^4]}$$ $\left[\because k = \frac{d_i}{d_o} \right]$

from this eqn we can find d_o & d_i.

Note!

(1) The hollow shafts are usualy used in marine work. These hollow shafts are stronger like solid shafts.

-> when a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same.

$$\therefore T \Rightarrow \cancel{\frac{\pi}{16}} \cdot \cancel{\tau} \cdot d_o^3 [1 - (k)^4] = \cancel{\frac{\pi}{16}} \cancel{\tau} \cdot d^3.$$

$$d_o^3 (1 - (k)^4) = d^3.$$

(2) The twisting moment (T) may obtained by using the following relation.

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

T = Twisting moment (N-m)

N = speed of the shaft (rpm)

(3) In case of belt drives, the twisting moment (T) is,

$$T = (T_1 - T_2)R$$

T_1 & T_2 = Tensions in tight side & slack side of the belt.

R = Radius of the pulley

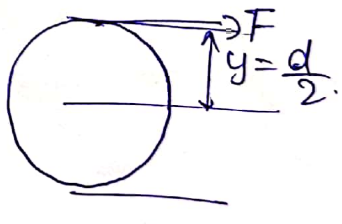
⟹ **Design of the shaft on the basis of bending moment only:**

→ Bending stress, $\sigma_b = \frac{M}{Z}$ → M = max. Bending moment (N-mm)

Z = section modulus (mm³)

for solid shaft:

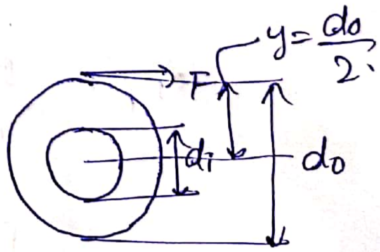
$$Z = \frac{I}{y} = \frac{\frac{\pi}{64} d^4}{d/2} \Rightarrow \frac{\pi}{\cancel{64}_{32}} \cancel{d^4}^{\,3} \times \frac{\cancel{2}}{\cancel{d}}$$



$$= \frac{\pi}{32} d^3.$$

for hollow shaft:

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}(d_o^4 - d_i^4)}{d_o/2}$$



$$= \frac{\pi}{\cancel{64}_{32}} \frac{d_o^4 - d_i^4}{d_o} \times \cancel{2}$$

$$Z = \frac{\pi}{32}\left[\frac{d_o^4 - d_i^4}{d_o}\right]$$

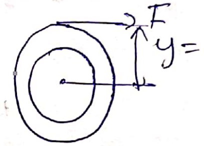
(A) for solid shaft:

$$\sigma_b = \frac{M}{Z} \Rightarrow \frac{M}{\frac{\pi}{32}d^3} \Rightarrow \frac{32M}{\pi d^3}$$

$$\sigma_b = \frac{32M}{\pi d^3}$$

$$\boxed{M = \frac{\pi}{32} d^3 \times \sigma_b}$$ → from this eqn we can calculate 'd'.

(B) for hollow shaft:



$$\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi}{32}\cancel{d_o^3}\left[\frac{d_o^4 - d_i^4}{d_o}\right]}$$

$$Z = \frac{\pi}{32}\left[\frac{d_o^4 - d_i^4}{d_o}\right] \Rightarrow \frac{\pi}{32}\,\frac{d_o^3}{d_o^3}\left[\frac{d_o^4 - d_i^4}{d_o}\right]$$

$$\Rightarrow \frac{\pi}{32} d_o^3 \left[\frac{d_o^4 - d_i^4}{d_o^4}\right]$$

$$\Rightarrow \frac{\pi}{32} d_o^3 \left[\frac{\cancel{d_o^4}}{\cancel{d_o^4}} - \frac{d_i^4}{d_o^4}\right]$$

$$= \frac{\pi}{32} d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right] \qquad \left[\because K = \frac{d_i}{d_o}\right]$$

$$= \frac{\pi}{32} d_o^3 (1 - K^4)$$

$$\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi}{32} d_o^3 (1 - K^4)}$$

$$\boxed{M = \frac{\pi}{32} d_o^3 (1 - K^4) \cdot \sigma_b}$$ → from this we can find d_o & d_i.

⇒ Design of the shaft on the Basis of combined Bending moment and Twisting moment:

When the shaft is subjected to combined Bending and twisting, then the shaft must be designed on the basis of the two moments simultaneously.

→ Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses.

→ The 2 theories are,

1) Maximum shear stress theory.
2) Principal " "

(i) Maximum shear stress theory:

We know that,

$$T = \frac{\pi}{16} \tau \cdot d^3$$

$$16T = \pi \cdot \tau \cdot d^3$$

$$\tau = \frac{16T}{\pi d^3} \rightarrow (1)$$

Also,

$$\sigma_b = \frac{32M}{\pi d^3} \rightarrow (2)$$

According to maximum shear stress theory, :

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_1^2 + 4\tau^2} \qquad \left[\because \text{here } \sigma_2 = 0,\ \sigma_1 = \sigma_b,\ \tau = \tau\right]$$

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \sqrt{\frac{1}{4}\left(\frac{32M}{\pi d^3}\right)^2 + \frac{1}{\cancel{4}}\cdot\cancel{4}\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \sqrt{\frac{1}{\cancel{4}}\times\left(\frac{\cancel{2}\times 16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \sqrt{\left(\frac{16}{\pi d^3}\right)^2 \left[M^2 + T^2\right]}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\frac{\pi}{16} d^3 \cdot \tau_{max} = \sqrt{M^2 + T^2}$$

$$\frac{\pi}{16} \cdot d^3 \cdot \tau_{max} = T_e.$$

T_e = Equivalent twisting moment in N-m.

(2) Maximum normal stress theory!

we know that, $\tau = \frac{16T}{\pi d^3}$, $\sigma_b = \frac{32M}{\pi d^3}$.

$(\sigma_b)_{max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$ here $\sigma_2 = 0$

$\sigma_1 = \sigma_b$.

$$(\sigma_b)_{max} = \frac{1}{2}\frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\cdot\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{1}{2}\frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{2\times 16T}{\pi d^3}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{32M}{\pi d^3} + \frac{1}{2}\cdot\frac{32}{\pi d^3}\sqrt{M^2+T^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\cdot M + \sqrt{M^2+T^2}\right].$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right].$$

$$(\sigma_b)_{max} = \frac{32}{\pi d^3}[M_e].$$

$$\frac{\pi}{32}d^3\cdot(\sigma_b)_{max} = M_e.$$

$$M_e = \frac{\pi}{32}d^3\cdot(\sigma_b)_{max}$$

M_e = equivalent Bending moment in N-m.

→ for hollow shafts, $T_e = \sqrt{M^2+T^2} = \frac{\pi}{16}\cdot\tau\cdot(d_o)^3(1-k^4)$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right) = \frac{\pi}{32}\times\sigma_b\,(d_o)^3(1-k^4)$$

→ Design of the shafts subjected to fluctuating loads:

→ In practical situations, the shafts are subjected to fluctuating torque and bending moments.

→ In order to design shafts subjected to fluctuating torque & Bending moment, combined shock and fatigue factors must be taken into account for the calculation of T_e & M_e.

We know that

T_e = equivalent twisting moment in N-m.

$$T_e = \sqrt{M^2 + T^2}$$

M_e = Equivalent Bending moment (N-m)

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

But for fluctuating loads,

$$\boxed{T_e = \sqrt{(K_m \cdot M)^2 + (K_t \cdot T)^2}}$$

$$\boxed{M_e = \frac{1}{2}\left[K_m \cdot M + \sqrt{(K_m \cdot M)^2 + (K_t \cdot T)^2}\right]}$$

K_m = Combined shock and fatigue factor for bending

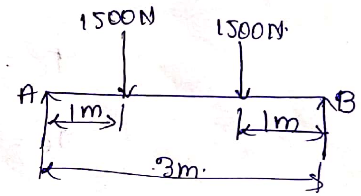
K_t = " " " " " " twisting

$$\therefore \boxed{T_e = \frac{\pi}{16} \cdot d^3 \cdot \tau_{max}}$$

$$\boxed{M_e = \frac{\pi}{32} \cdot d^3 \cdot (\sigma_b)_{max}}$$

Q) A shaft made up of mild-steel is required to transmit 100 kW at 300 rpm. The supported length of the shaft is 3 m. It carries two pulleys each weighing 1500 N supported at a distance of 1 m from the ends respectively. Assuming safe value of stress as 60 MPa, determine the dia of the shaft.

Sol: Given data:



$P = 100\,kW = 100\times10^3\,W$.

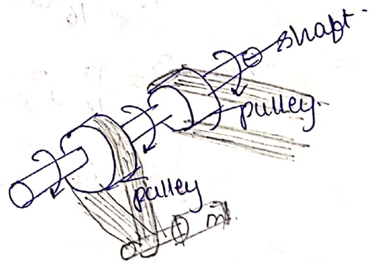
$N = 300\,rpm$.

$W = 1500\,N$.

Safe stress $= 60\,MPa$.

$d = ?$

Type = solid shaft.



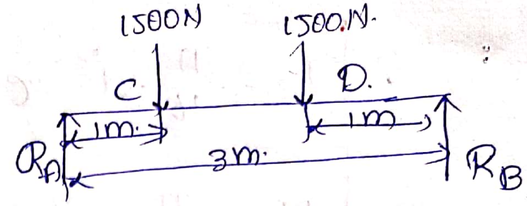
The given shaft is subjected to both twisting & Bending.

$\therefore$ The shaft will be designed based on TM & BM.

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P\times60}{2\pi N} = \frac{100\times10^3\times60}{2\pi\times300} = 3183\,N\text{-}m = 3183\times10^3\,N\text{-}mm.$$

So, It is a simply supported beam.



$R_A + R_B = 3000$ N.

$M_A = 0$,

$R_B \times 3 = 1500 \times 2 + 1500 \times 1$

$3R_B = 3000 + 1500$

$R_B = \frac{4500}{3} = 1500$

$\therefore R_A + R_B = 3000$ then

$R_A = 1500$ N.

$R_B = 1500$ N

BM at 'C' : BCZ Beam is symmetric

$M_C = 1500 \times 1$

$= 1500$ N-m $= 1500 \times 10^3$ N-mm.

Equivalent Twisting moment,

$$T_e = \sqrt{T^2 + M^2}$$

$$T_e = \sqrt{(3183)^2 + (1500)^2}$$

$$T_e = 3519 \text{ N-m} \Rightarrow 3519 \times 10^3 \text{ N-mm}$$

$$T_e = \frac{\pi}{16} d^3 \tau \Rightarrow$$

$$3519 \times 10^3 = \frac{\pi}{16} d^3 \times 60$$

$$\boxed{d = 66.8 \approx 70\text{mm.}}$$

(Q) A shaft is supported by two bearings placed 1m apart. A 600mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having max tension of 2.25 KN. Another pulley 400 mm dia. is placed 200mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulley is 180° and $\mu = 0.24$. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Sol:- Let T_1 = Tension in tight side of belt on pulley C.

$T_1 = 2250 N$.

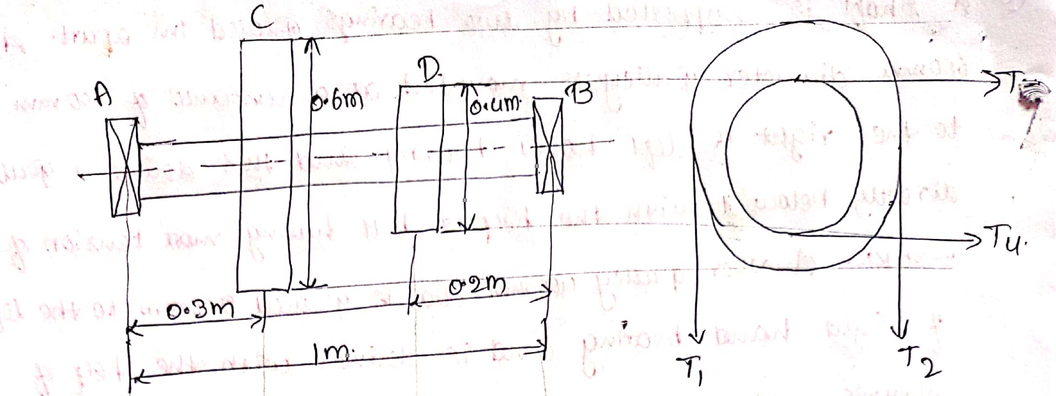
T_2 = Tension " slack " " "

Angle of contact of belts on both the pulleys, $\theta = 180°$

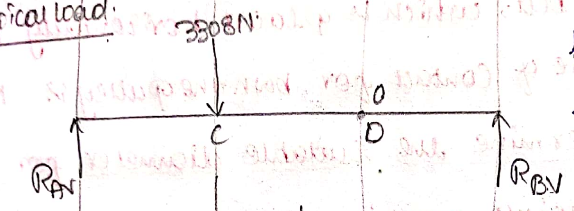
Coefficient of friction $\mu = 0.24$

Allowable working stress in tension, $\sigma_t = 63 MPa = 63 N/mm^2$

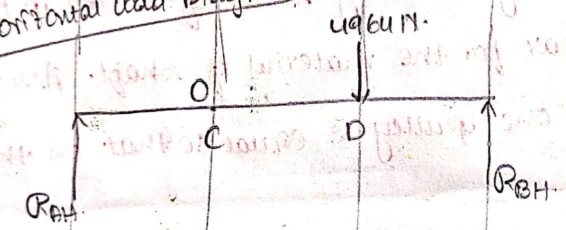
" " in shear, $\tau = 42 MPa = 42 N/mm^2$



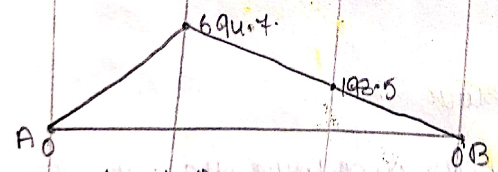
Vertical load:



Horizontal load Diagram:



Max Bm for vertical loading:



Bm Diagram for Horizontal load:

here,

$T_1 = 2250\ N.$

$\frac{T_1}{T_2} = e^{\mu\theta}$

$\frac{2250}{T_2} = e^{0.24\times\pi}$ $\left[\because \theta = \frac{\pi}{180}\times 180 = \pi\ rad\right]$

$T_2 = 1058N.$

vertical load acting on the shaft at $D = 0$

vertical load acting on the shaft $C = T_1 + T_2$

$= 2250 + 1058$

$= 3308N.$

$\Rightarrow$ To find horizontal loads:-

horizontal load at $C = 0$

horizontal load at $D = T_3 + T_4$

$\frac{T_3}{T_4} = e^{\mu\theta}$

$\frac{T_3}{T_4} = e^{0.24\times\pi} \Rightarrow \frac{T_3}{T_4} = 2.125 \rightarrow (2)$

But Torque (T) on pulley C & D are same.

$\therefore T \Rightarrow (T_1 - T_2) R_C = (T_3 - T_4) R_D$

$T = (T_1 - T_2) R_C$

$= (2250 - 1058) \times 0.3$

$\boxed{T = 357.6 \text{ N-m}}$

$T = (T_3 - T_4) \times R_D$

$357.6 = (T_3 - T_4) \times 0.2$

$T_3 - T_4 = 1788 \text{ N} \longrightarrow (2) \Rightarrow T_3 = 1788 + T_4$

solve eqn (1) & (2)

$\frac{T_3}{T_4} = 2.125$

$\frac{\cancel{T_3}}{\cancel{1788}+T_4} \quad \frac{1788 + T_4}{T_4} = 2.125$

$T_3 = 3376 \text{ N}, \quad T_4 = 1588 \text{ N}$

$\therefore$ horizontal load acting on the shaft at D

$D = T_3 + T_4$

$= 3376 + 1588$

$= 4964 \text{ N}$

find the max. Bending moment for vertical loading.

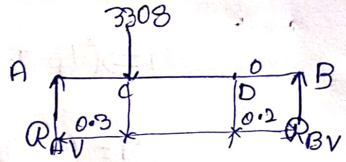
→ from vertical loading diagram,

$R_{AV} + R_{BV} = 3308 \text{ N}$

Scanned with CamScanner

→ Taking moments about 'A'

$\Sigma M_A = 0,$



$R_{BV} \times 1 - 3308 \times 0.3 = 0$

$R_{BV} = 3308 \times 0.3$

$\underline{R_{BV} = 992.4 \text{ N}.}$

$R_{AV} = 3308 - R_{BV}$

$= 3308 - 992.4$

$\underline{R_{AV} = 2315.6 \text{ N}}$

→ Now find the max Bending moment,

$BM_{@BV} = 0$ (Bcz it is simply supported Beam)

$BM_{@D} = R_{BV} \times 0.2$

$= 992.4 \times 0.2 = 198.48 = 198.5 \text{ N-m}$

$BM_{@C} = R_{BV} \times 0.7 = 992.4 \times 0.7 = 694.7 \text{ N-m}.$

$BM_{@AV} = 0.$

⇒ find the max. B.m for horizontal loading,

$R_{AH} + R_{BH} = 4964 \text{ N}.$

taking moments about 'A'.

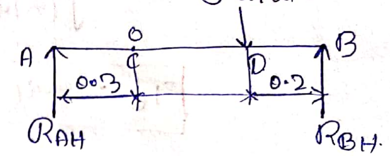
$R_{BH} \times 1 = 4964 \times 0.8$

$R_{BH} = 3971 \text{ N}.$

$R_{AH} = 4964 - R_{BH}$

$R_{AH} = 4964 - 3971 = 993 \text{ N}.$

Now find the max Bm for horizontal loading:



$BM@_{BH} = 0$

$BM@D = R_{BH} \times 0.2$

$= 3971 \times 0.2 = \underline{794.2}$ N-m

$BM@C = R_{BH} \times 0.7 - 4964 \times 0.5$

$= 3971 \times 0.7 - 4964 \times 0.5$

$= 297.9$ N-m

$BM@_{AH} = 0$

→ Resultant BM at C

$$M_C = \sqrt{(BM_{CV})^2 + (BM_{CH})^2}$$

$$= \sqrt{(694.7)^2 + (297.9)^2}$$

$= 756$ N-m

Resultant Bm at D.

$$M_D = \sqrt{(BM_{DV})^2 + (BM_{DH})^2}$$

$$= \sqrt{(198.5)^2 + (794.2)^2}$$

$= 819.2$ N-m.

∴ Bending moment is maximum at 'D'.

Max. Bm, $m = 819.2$ N-m.

Torque $T = 357.6$ N-m.

∴ equivalent twisting moment

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(819.2)^2 + (357.6)^2}$$

$$\boxed{T_e = 894 \text{ N-m}} = 894 \times 10^3 \text{ N-mm}$$

we have,

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

$$894 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$$

$$\boxed{d = 47.6 \text{ mm}}$$

→ equivalent Bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

$$= \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(819.2 + 894)$$

$$= 856.6 \cancel{\times 10} \text{ N-m}$$

$$\boxed{M_e = 856.6 \times 10^3 \text{ N-mm}}$$

we have,

$$M_e = \frac{\pi}{32} \times \sigma_b \times d^3$$

$$856.6 \times 10^3 = \frac{\pi}{32} \times 63 \times d^3$$

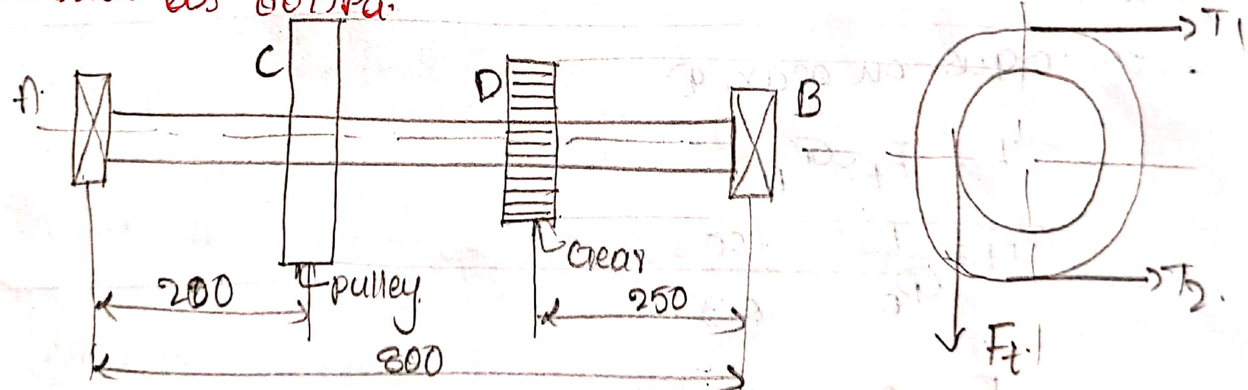
$$\boxed{d = 51.7 \text{ mm}}$$

∴ Taking larger 'd' of two values.

i.e. $d = 51.7$ mm $\approx$ 55mm

$$\boxed{d = 55 \text{ mm}}$$

Q: Determine the diameter of solid shaft for the following case.
The shaft is supported in bearings which are at 800mm apart. A pulley of 700mm dia. is placed 200mm right of the left hand bearing 'A' and a gear of 600mm pitch diameter is placed at a distance of 250mm towards left of right hand bearing B. The gear is of 20° straight tooth spur gear. The gear is driven by down ward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The max. belt tension is 3000 N. and the tension ratio is 3:1. Assume that torque is same on pulley and gear. Take allowable shear stress for the shaft material as 40 MPa and allowable stress in tension as 60MPa.



Sol) Given:

Let max. Tension in the belt, $T_1 = 3000$ N.
min. " " ", T_2.

Ratio of belt tensions. $\frac{T_1}{T_2} = \frac{3}{1}$

$T_2 = 1000$ N.

Diameter of the pulley, $C = 700$mm, $R_C = 350$mm $= 0.35$m
" " ", $D = 600$mm, $R_D = 300$mm $= 0.3$m.

Pressure angle for gear,

$\phi = 20^\circ$

Angle of wrap of the belt, $\theta = 180^\circ$

weight of the pulley, $W = 2000\ N$.

$\sigma = 60\ MPa = 60\ N/mm^2$

$\tau = 40\ MPa = 40\ N/mm^2$

→ Torque acting on pulley C,

$T = (T_1 - T_2) \times R_C$.

$= (3000 - 1000) \times 0.35$

$T = 700\ N\text{-}m$.

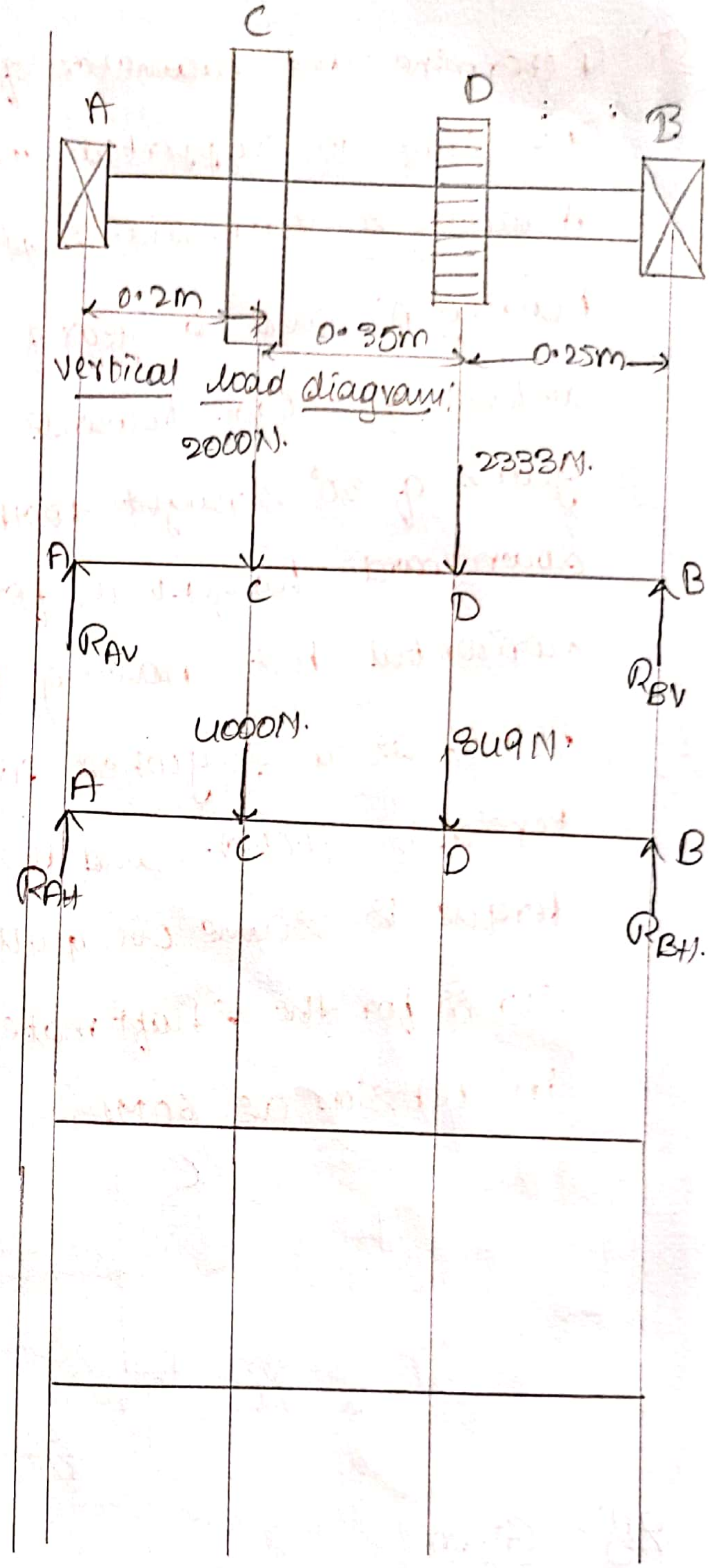
It is given that torque acting on the pulley and gear is same.

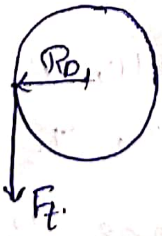
∴ So, torque on gear, D.

$T = F_t \times R_D$.

$F_t = \frac{T}{R_D} = \frac{700}{0.3}$

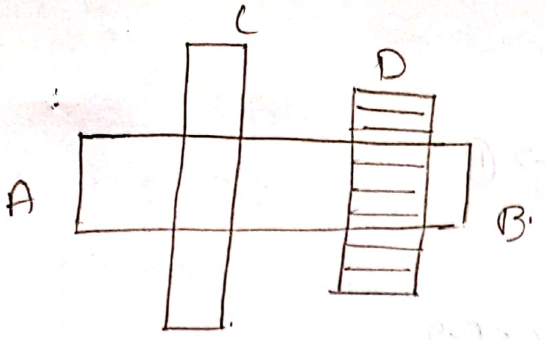
$F_t = 2333\ N$.

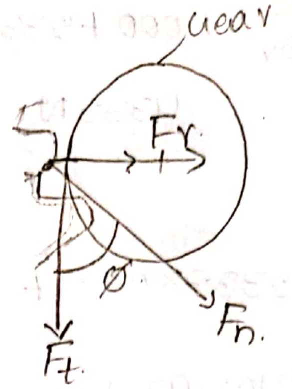


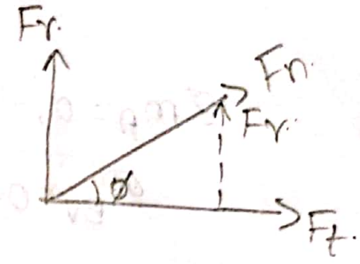


F_t will be the vertical load on shaft at 'D'

∴ vertical load on shaft at D = 2333 N.







∴ from the diagram,

horizontal load on shaft at D will be 'Fr'.

$F_r = F_n \sin\phi$ $\qquad \left(\because \sin\phi = \frac{F_r}{F_n}\right)$

To find F_n,

$$\cos\phi = \frac{F_t}{F_n}$$

$$F_n = \frac{F_t}{\cos\phi} = \frac{2333}{\cos(20°)} = 2482.72$$

$F_n = 2483\ N$.

$F_r = F_n \sin\phi$

$F_r = 2483 \times \sin 20° = 849\ N$.

∴ horizontal load on shaft at 'D', $= F_r = 849\ N$.

⇒ vertical load on the shaft 'C' = weight of the pulley

$= 2000\ N$.

horizontal load on the shaft 'C' $= T_1 + T_2$

$= 3000 + 1000$

$= 4000\ N$.

To find the vertical Bending moments:

from vertical loading diagram,

$R_{AV} + R_{BV} = 2000 + 2333$

$= 4333\,N \longrightarrow (1)$

$\Sigma m_A = 0,$

$R_{BV} \times 0.8 = 2333 \times 0.55 + 2000 \times 0.2$

$R_{BV} = 2103.93\,N$

from eqn (1), $R_{AV} = 4333 - R_{BV}$

$= 4333 - 2104$

$R_{AV} = 2229\,N.$

Now:

$BM_{BV} = 0$

$BM@_{DV} = R_{BV} \times 0.25 = 2104 \times 0.25 = 526\,N\text{-}m$

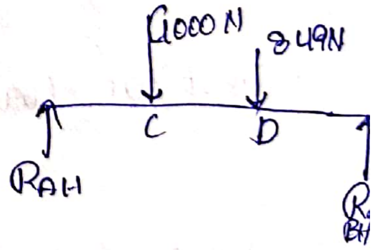
$BM@_{CV} = R_{BV} \times 0.6 - 2333 \times 0.35 = 2104 \times 0.6 - 2333 \times 0.35$

$= 445.81\,N\text{-}m.$

$BM@_{AV} = 0$

To find the horizontal Bending moment,

from horizontal loading diagram



$R_{AH} + R_{BH} = 4000 + 849$

$= 4849\,N. \longrightarrow (2)$

$\Sigma m_A = 0.$

$R_{BH} \times 0.8 = 849 \times 0.55 + 4000 \times 0.2$

$R_{BH} = 1583.68. \approx 1584\,N.$

from eqn (2), $R_{AH} = 4849 - R_{BH}$

$= 4849 - 1584$

$R_{AH} = 3265\,N.$

Now,

$BM@_{BH} = 0$

$BM@_{DH} = R_{BH} \times 0.25 = 1584 \times 0.25 = 396$ N-m.

$BM@_{CH} = R_{AH} \times 0.2 = 3265 \times 0.2 = 653$ N-m.

$BM@_{AH} = 0$

→ Resultant Bending moment at 'D'

$$M_D = \sqrt{(BM_{DV})^2 + (BM_{DH})^2}$$

$$= \sqrt{(526)^2 + (396)^2}$$

$$= \sqrt{433492}$$

$$M_D = 658.4 \text{ N-m}$$

Resultant BM @ 'C'

$$M_C = \sqrt{(BM_{CV})^2 + (BM_{CH})^2}$$

$$= \sqrt{(445.8)^2 + (653)^2}$$

$$M_C = 790.66 \text{ N-m}$$

∴ Resultant BM at 'C' is more than 'D'.

So, max. Bending moment on the shaft

= Resultant Bm at 'C'

$$M = M_C = 790.66 \text{ N-m.}$$

$$M = 790.66 \times 10^3 \text{ N-mm}$$

Torque on shaft $T = 700$ N-m

$$T = 700 \times 10^3 \text{ N-mm}$$

Equivalent twisting moment.

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(790.66)^2 + (700)^2}$$

$$T_e = 1056 \text{ N-m.}$$

$$T_e = 1056 \times 10^3 \text{ N-mm}$$

we have,

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

$$1056 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$$\boxed{d = 51.23 \text{ mm}}$$

Equivalent Bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{T^2 + M^2}\right)$$

$$= \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(790.66 + 1056)$$

$$= 923.33 \text{ N-m}$$

$$M_e = 923.33 \times 10^3 \text{ N-mm.}$$

we have,

$$M_e = \frac{\pi}{32} \times \sigma \times d^3$$

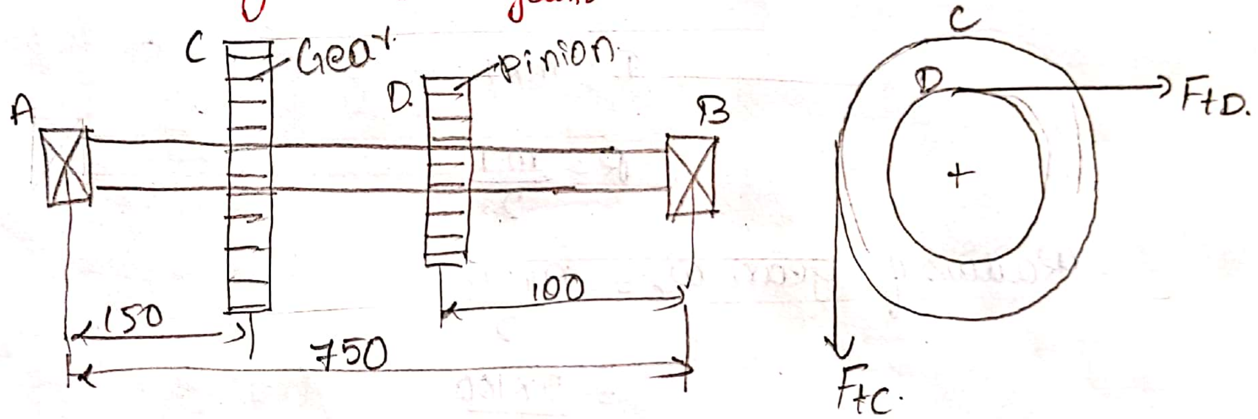
$$923.33 \times 10^3 = \frac{\pi}{32} \times 60 \times d^3$$

$$\boxed{d = 53.91 \text{ mm.}}$$

Taking larger value of two, $d = 53.91$ mm

$$\therefore \boxed{d = 55 \text{ mm.}}$$

6. Determine the dia of solid shaft for the following case. The shaft is transmitting 15 kW power at 200 rpm. It is supported on two bearings 750 mm apart and has two gears keyed to it. (as shown in fig.) The pinion having 30 teeth of 5 mm module delivers power horizontally. The gear having 100 teeth of 5 mm module receives power in vertical direction. Take allowable stress in tension as 80 MPa and in shear as 54 MPa. Assume same torque is acting on both gears.



Sol:- Given data:

Power, $P = 15\,kW = 15 \times 10^3\,W$.

Speed of shaft, $N = 200\,rpm$.

no. of teeth on pinion D, $T_D = 30$

module of pinion, $m_D = 5\,mm$

no. of teeth on gear, C : $T_C = 100$

module of gear C = $m_C = 5\,mm$

Allowable stress in tension, $\sigma_t = 80\,MPa$

$= 80\,N/mm^2$

A

in shear, $\tau = 54\,MPa$

$= 54\,N/mm^2$

Torque transmitted by the shaft,

$$P = \frac{2\pi NT}{60}, \quad T = \frac{60P}{2\pi N}$$

$$= \frac{15\times10^3\times60}{2\pi\times200}$$

$$T = 716\ N\text{-}m$$

$$T = 716\times10^3\ N\text{-}mm.$$

from gears, module, $m = \frac{D}{T}$

$$D = m\times T.$$

$$R = \frac{m \cdot T}{2}.$$

$\therefore$ Radius of gear, $R_c = \frac{m_c \cdot T_c}{2}.$

$$= \frac{5\times100}{2}$$

$$R_c = 250\ mm.$$

Radius of pinion, $R_D = \frac{m_D T_D}{2}$

$$= \frac{5\times30}{2}$$

$$R_D = 75\ mm$$

Tangential force acting on the gear, C:

$$T = F_{tc}\times R_c.$$

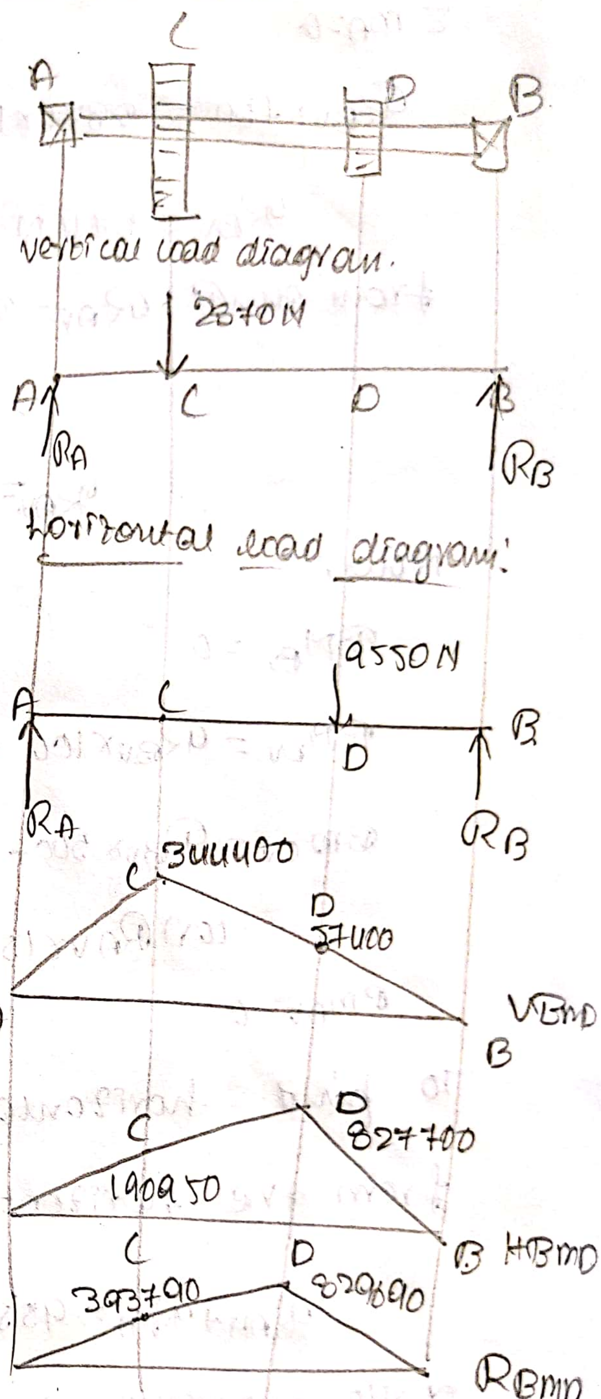
$$F_{tc} = \frac{T}{R_c}$$

$$= \frac{716\times10^3}{250}$$

$$F_{tc} = 2870\ N.$$

F_{tc}.

This tangential force will be the vertical load on the shaft at 'C'.



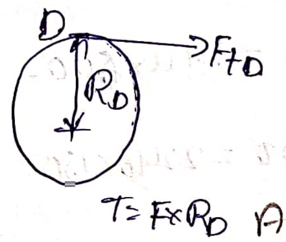
∴ vertical load acting on the shaft at $C = 2870\,N$.

There is no horizontal load (force) on gear.

∴ Horizontal load acting on the shaft at $C = 0$

→ Tangential force acting on the gear, D

$$F_{tD} = \frac{T}{R_D}$$



$$= \frac{716 \times 10^3}{75}$$

$$F_{tD} = 9550\,N.$$

→ This tangential load will be the horizontal load acting on shaft at D.

∴ horizontal load acting on shaft at $D = 9550\,N$.

vertical load on shaft $D = 0$.

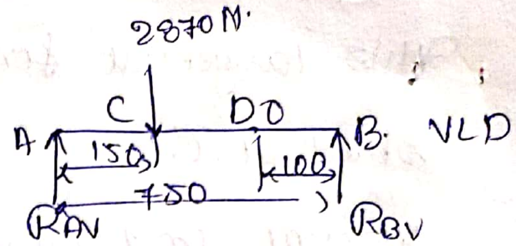
→ find the Bending moments.

To find the vertical Bending moments:

from the vertical loading diagram.

$$R_{AV} + R_{BV} = 2870\,N \quad \rightarrow ⓑ$$

Take moments about point A = 0



$\Sigma M_A = 0$

$R_{BV} \times 750 = 2870 \times 150$

$R_{BV} = 574$ N.

from eqn ①, $R_{AV} = 2870 - R_{BV}$

$= 2870 - 574$

$R_{AV} = 2296$ N

Now,

$BM_{BV} = 0$

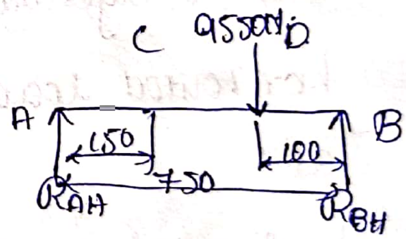
$BM_{DV} = R_{BV} \times 100 = 574 \times 100 = 57400$ N-mm.

$BM_{CV} = R_{BV} \times 600 = 574 \times 600 = 344400$ N-mm

(or) $R_{AV} \times 150 = 2296 \times 150 = 344400$ N-mm.

$BM_{AV} = 0$

→ To find horizontal Bending moments from the horizontal loading diagram.



$R_{AH} + R_{BH} = 9550$ N. →②

Take moments about point A = 0

$R_{BH} \times 750 = 9550 \times 650$

$R_{BH} = 8277$ N.

from eqn ② $R_{AH} = 9550 - R_{BH}$

$= 9550 - 8277$

$R_{AH} = 1273$ N

Now,

$BM@_{BH} = 0.$

$BM@_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827700$ N-mm

$BM@_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190950$ N-mm

$BM_{AH} = 0$

Resultant Bending moment at 'D':

$$M_D = \sqrt{(BM_{DV})^2 + (BM_{DH})^2}$$

$$= \sqrt{(57400)^2 + (827700)^2}$$

$$M_D = 829690 \text{ N-mm}$$

Resultant BM at 'C'

$$M_C = \sqrt{(BM_{CV})^2 + (BM_{CH})^2}$$

$$= \sqrt{(344400)^2 + (190950)^2}$$

$$M_C = 393790 \text{ N-mm.}$$

∴ max BM on the shaft = Resultant BM at D.

$$M = M_D = 829690 \text{ N-mm.}$$

→ Now equivalent Twisting moment,

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(829690)^2 + (716 \times 10^3)^2}$$

$$T_e = 1096 \times 10^3 \text{ N-mm.}$$

we have, $T_e = \frac{\pi}{16} \times d^3 \times \tau.$

$$T_e = \frac{\pi}{16} \times d^3 \times \tau$$

$$1096 \times 10^3 = \frac{\pi}{16} \times 54 \times d^3$$

$$\boxed{d = 47mm.}$$

→ Equivalent Bending moment.

$$M_e = \frac{1}{2}\left(M + \sqrt{T^2 + M^2}\right)$$

$$= \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(829690 + 1096 \times 10^3)$$

$$M_e = 962845 \text{ N-mm}$$

But $M_e = \frac{\pi}{32} \times \sigma_b \times d^3$

$$962845 = \frac{\pi}{32} \times 80 \times d^3$$

$$\boxed{d = 49.67mm} \approx 50mm.$$

∴ Diameter of one shaft, $\boxed{d = 50\, mm.}$

→ Shafts Subjected to Axial load in addition to Combined Torsion and Bending loads:

→ This combination of loads acts in propeller shafts of ships and shafts for driving worm gears, then stress due to axial load must be added to the bending stress(σ_b).

→ we know, bending stress, $\sigma_b = \frac{32M}{\pi d^3}$.

→ stress due to axial load, $\sigma = \frac{F}{A} = \frac{F}{\frac{\pi}{4}d^2} = \frac{4F}{\pi d^2}$ (for solid)

→ for hollow shafts, $\sigma_a = \frac{4F}{\pi[(d_o)^2 - (d_i)^2]}$

$$= \frac{4F}{\pi d_o^2\left[\frac{d_o^2}{d_o^2} - \frac{d_i^2}{d_o^2}\right]}$$

$$= \frac{4F}{\pi (d_o)^2\left[1 - \left(\frac{d_i}{d_o}\right)^2\right]}.$$

$$= \frac{4F}{\pi (d_o)^2 (1-k^2)} \quad \left[\because \frac{d_i}{d_o} = k\right].$$

∴ Total normal stress,

$$\sigma = \sigma_a + \sigma_b.$$

$$= \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$$

$$= \frac{32M}{\pi d^3} + \frac{8}{8}\left(\frac{4F}{\pi d^2}\right)$$

$$= \frac{32M}{\pi d^3} + \frac{32F}{8\pi d^2}$$

$$= \frac{32}{\pi d^3}\left[M + \frac{F\times d}{8}\right].$$

$$\sigma = \frac{32M_1}{\pi d^3} \qquad \left[\because M_1 = M + \frac{F\times d}{8}\right].$$

In case of hollow shaft,

$$\sigma = \sigma_b + \sigma_a.$$

$$= \frac{32M}{\pi (d_o)^3(1-k^4)} + \frac{4F}{\pi (d_o)^2(1-k^2)}$$

$$= \frac{32M}{\pi (d_o)^3(1-k^4)} + \frac{8}{8}\,\frac{4F}{\pi (d_o)^2(1-k^2)}$$

$$= \frac{32}{\pi (d_o)^3(1-k^4)}\left[M + \frac{F d_o(1+k^2)}{8}\right].$$

$$\frac{32F}{8\pi(d_o)^2(1-k^2)} \times \frac{(1+k^2)}{(1+k^2)}$$

$$\Rightarrow \frac{32F\times(1+k^2)}{8\pi(d_o)^2(1+k^2-k^2-k^4)} \times \frac{d_o}{d_o}$$

$$= \frac{32F\times(1+k^2)\times d_o}{8\pi(d_o)^3(1-k^4)}$$

$$\boxed{\sigma = \frac{32M_1}{\pi (d_o)^3(1-k^4)}}$$

$$M_1 = \left[M + \frac{F d_o(1+k^2)}{8}\right].$$

→ In case of long shafts subjected to compressive loads, a factor known as column factor (α) must be introduced to take the column effect into account.

$\left[\because \text{generally if } F = \text{axial stress} \quad \alpha = 1.\right]$

∴ stress due to the compressive load,

$$\sigma_c = \frac{\alpha\times 4F}{\pi d^2} \text{ (for solid)}, \quad \sigma_c = \frac{\alpha\times 4F}{\pi(d_o)^2(1-k^2)} \text{ (hollow)}$$

→ So, if any shaft subjected under axial, Bending and Twisting load then max principal & max shear stress will be,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma)^2 + 4\tau^2}$$

here, $\sigma = \sigma_a + \sigma_b$.

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M_1}{\pi(d_o)^3(1-K^4)}\right)^2 + 4\left(\frac{16T}{\pi(d_o)^3(1-K^4)}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{32M_1}{\pi(d_o)^3(1-K^4)}\right)^2 + \frac{\cancel{4}}{\cancel{4}}\left(\frac{2\times16T}{\pi(d_o)^3(1-K^4)}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{32M_1}{\pi(d_o)^3(1-K^4)}\right)^2 + \left(\frac{32T}{\pi(d_o)^3(1-K^4)}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{32}{\pi(d_o)^3(1-K^4)}\right)^2 \cdot (M_1^2+T^2)}$$

$$\tau = \frac{1}{\cancel{2}} \times \frac{\overset{16}{\cancel{32}}}{\pi(d_o)^3(1-K^4)}\sqrt{M_1^2+T^2}$$

$$\frac{\pi}{16}(d_o)^3(1-K^4)\,\tau = \sqrt{M_1^2+T^2}. \qquad [\because T_e = \sqrt{M_1^2+T^2}].$$

$$\therefore T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o(1+K^2)}{8}\right]^2 + (K_t \times T)^2} \quad \text{(under fluctuating loads)}$$

$$\boxed{\frac{\pi}{16}(d_o)^3(1-K^4)\,\tau = \sqrt{\left[K_m \times M + \frac{\alpha F d_o(1+K^2)}{8}\right]^2 + (K_t \times T)^2}}$$

Similarly, for Me → equivalent Bending eqn.

$$M_e = \frac{1}{2}\left[M_1 + \sqrt{M_1^2 + T^2}\right].$$

$$\boxed{\frac{\pi}{32}\sigma_b (d_o)^3(1-K^4) = \frac{1}{2}\left[K_m \times M + \frac{\alpha \cdot F d_o (1+K^2)}{8} + \sqrt{\left(K_m \times M + \frac{\alpha F d_o (1+K^2)}{8}\right)^2 + (K_t \times T)^2}\right]}$$

(Q) A hollow shaft is subjected to a maximum torque of 1.5 KN-m. and a max. bending moment of 3 KN-m. It is subjected, at the same time, to an axial load of 10 KN. Assume that the load is applied gradually and the ratio of the inner dia to the outer dia. is 0.5. If the outer dia. of the shaft is 80mm, find the shear stress induced in the shaft.

Sol:- Given, $T = 1.5\,KN\text{-}m = 1.5 \times 10^3\,N\text{-}m.$

$M = 3\,KN\text{-}m = 3 \times 10^3\,N\text{-}m.$

$F = 10\,KN = 10 \times 10^3\,N.$

$K = \frac{d_i}{d_o} = 0.5$

$d_o = 80\,mm = 0.08\,m.$

$\tau = ?$

The load is gradually applied.

∴ from table, $K_m = 1.5$, $K_t = 1.$

we know $\alpha = 1.$ → for axial load.

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equivalent Twisting moment for hollow shaft, :

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1+K^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3\cancel{\times 10^3} \frac{1 \times 10 \times 10^3 \times (1+0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

$$= \sqrt{(4500+125)^2 + (1500)^2}$$

$$= 4862 \text{ N-m.}$$

$$T_e = 4862 \times 10^3 \text{ N-mm}$$

But, $T_e = \frac{\pi}{16} \tau (d_o)^3 (1-K^4)$

$$\frac{\pi}{16} \tau (d_o)^3 (1-K^4) = 4862 \times 10^3 \text{ N-mm}$$

$$\frac{\pi}{16} \times \tau \times (80)^3 (1-0.5^4) = 4862 \times 10^3$$

$$94260 \, \tau = 4862 \times 10^3$$

$$\tau = \frac{4862 \times 10^3}{94260}$$

$$\tau = 51.6 \text{ N/mm}^2 = 51.6 \text{ MPa.}$$

$$\therefore \boxed{\tau = 51.6 \text{ MPa.}}$$

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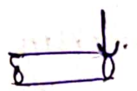
⇒ Design of shaft Based on rigidity!

Some times the shafts are to be designed on the basis of rigidity, so now we shall consider the following two types of Rigidity.

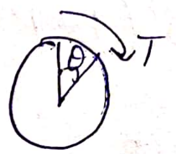
1) Torsional Rigidity.
2) Lateral Rigidity.

1) Torsional Rigidity!

→ Rigidity is, to avoid deformation (or) deflection for the applied load.



→ So, here we are dealing with the shaft for the applied torque the shaft will be twisted. so here, we are limit the angle of twist



If $\theta \downarrow$ → then rigidity is $\uparrow$

→ so take θ into consideration then find the dia. of the shaft.

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{J \cdot G}$$

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θ = Torsional deflection (or) angle of twist

T = Twisting moment.

J = Polar M.I. $= \frac{\pi}{32} d^4$ → solid shaft

$= \frac{\pi}{32}(d_o^4 - d_i^4)$ → hollow shaft.

G = Modulus of rigidity.

L = length of the shaft.

→ Torsional rigidity is important in the case of cam shaft of an I.C engine where the timing of the values would be effected.

→ Permissible amount of twist should not exceed $0.25°$ per meter length.

→ for line shafts $\theta = 2.5 - 3°/m$ length.

(2) Lateral rigidity:

It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of balance forces.

→ If the shaft is uniform c/s then lateral deflection of a shaft may obtained by, $\boxed{\delta = \frac{Pl}{AE}}$, $\sigma = \frac{P}{A}$, $\delta = \frac{Dl}{l}$, $E = \frac{\sigma}{\varepsilon}$, Hence $l = \frac{Pl}{AE}$.

→ If the shaft is variable c/s, then lateral deflection,

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}}$$